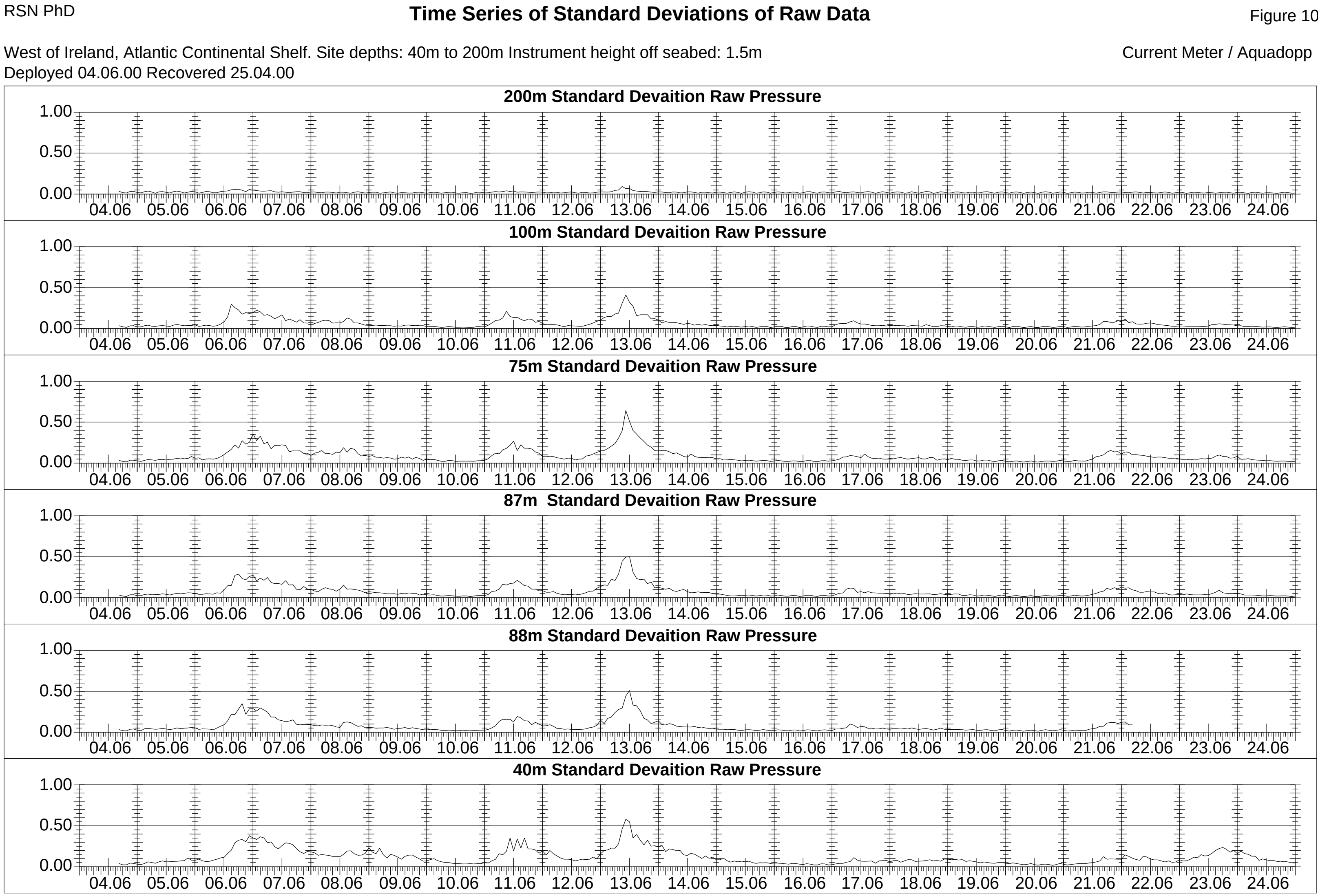
Wave measurement:
is a pressure sensor ever the right choice?

A comparative investigation of heave based measurements, acoustic surface tracking and pressure based measurements

Robin Newman, Emu Limited

Why am I asking this?



Measurements West of Ireland in 2000 taken using Seabed Aquadopps

- Simple Standard Deviation of the raw data from 6 sites shows pressure signal to 100m, Linear Wave theory does not support this

Why am I asking this?

RSN PhD

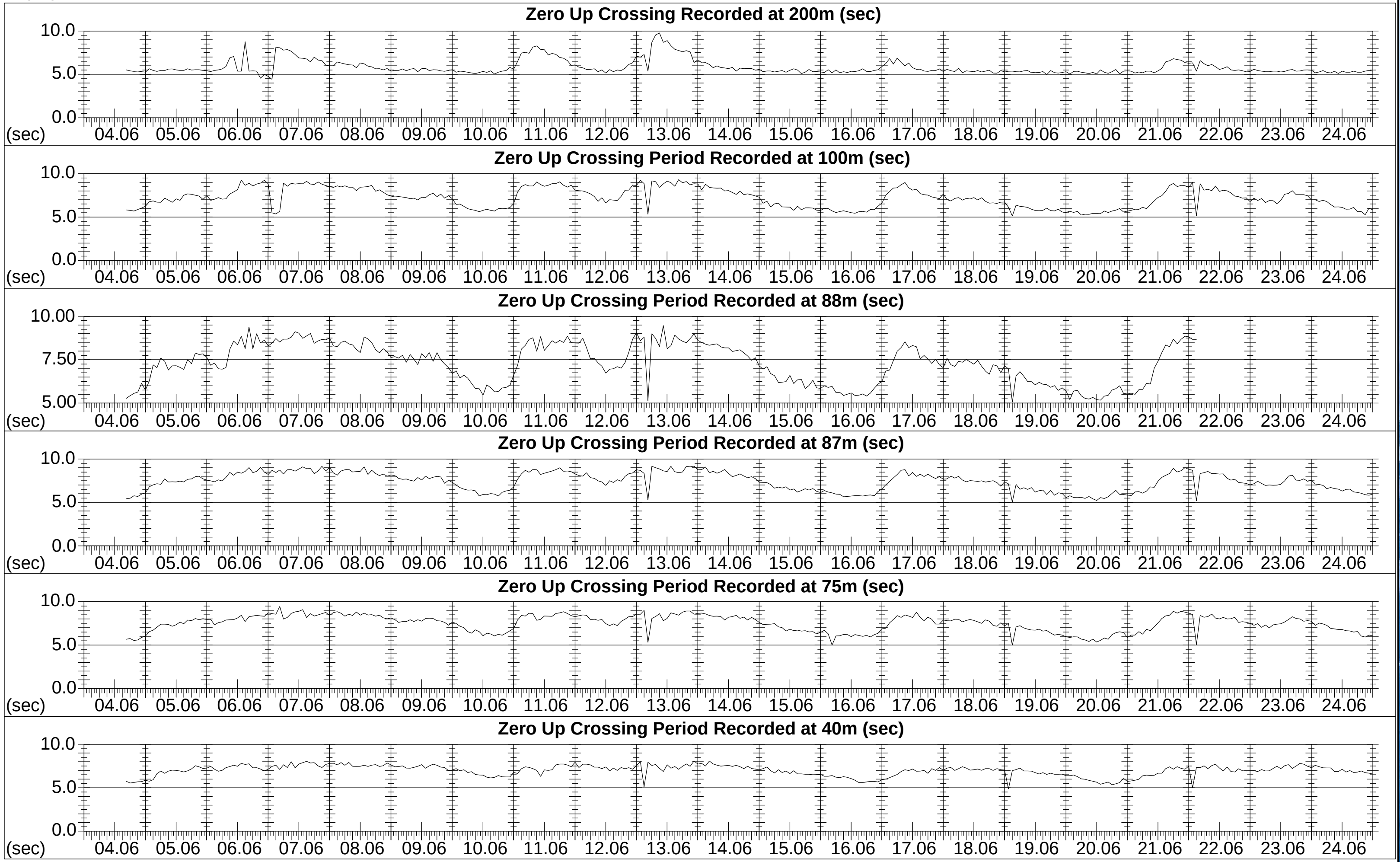
Time Series of Calculated Zero Up Crossing Periods

Figure 11

West of Ireland, Atlantic Continental Shelf. Site depths: 40m to 200m Instrument height off seabed: 1.5m

Current Meter / Aquadopp

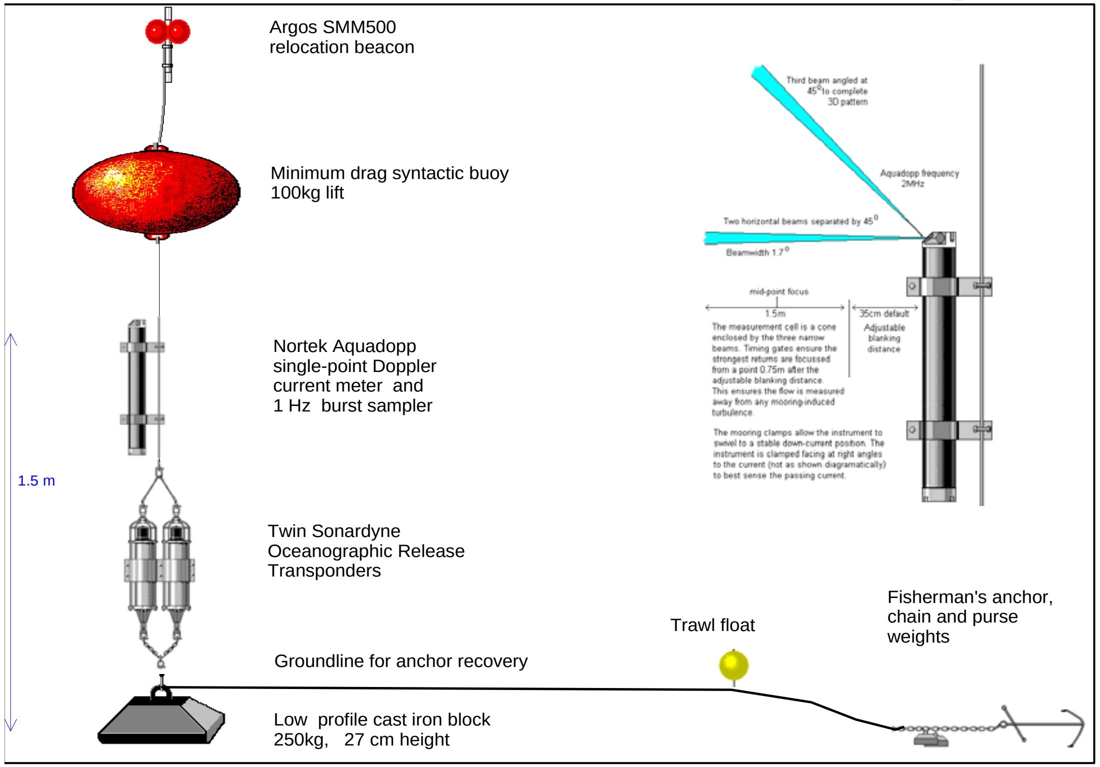
Deployed 04.06.00 Recovered 25.04.00



Measurements West of Ireland in 2000 taken using Seabed Aquadopps

- Deterministic Zero Up Crossing Periods also show consistency across the sites
- Wave periods in excess of 5 seconds and up to 10 seconds

How was the data acquired?



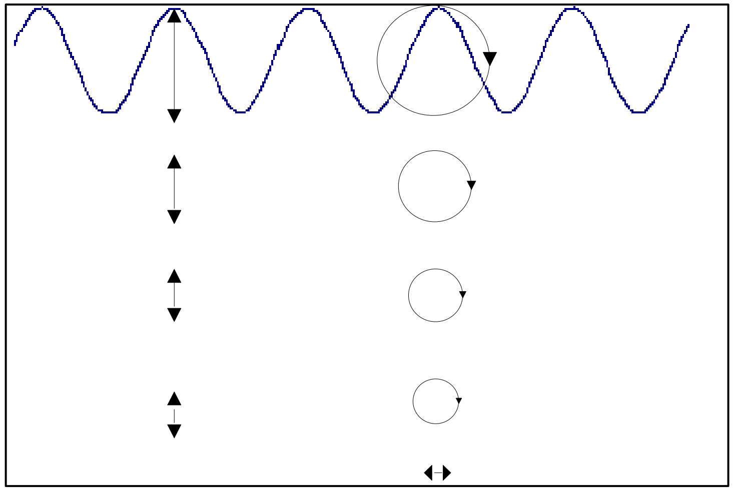
6 moorings deployed at depths from 40 to 200m

- All instruments 1.5m above the seabed
- Set to record tides and currents as a 40 second burst every 10 minutes
- QC burst set to record 513 1Hz records every hour (thus 512 available records)

Linear wave theory applied to the pressure data

$$P_s = \left[\frac{\cosh kh}{\cosh k(h+z)}\right]^2 \frac{P_d}{\rho^2 g^2}$$

P_s = pressure signal at surface
P_d = pressure signal at sensor
k = wave number
h = mean depth of burst
z = sensor height above seabed
ρ = water density
g = local gravity



Application of Linear Wave Theory

The wave number (k) is derived from the following equations:

$$c2 = gk \tanh kh$$

Where:

c = wave speed (celerity)
k = wave number
h = mean depth of burst
g = local gravity

Using the relationships:

$$k = \frac{2\pi}{L}$$

And

$$c = \frac{L}{T}$$

the derivation of L can be used.

$$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi}{L}\right)$$

Where:

L = wavelength   T = wave period

It is not possible to directly solve this equation therefore it is solved by iteration taking the deep water value for L and then iterating until the answer stabilises. The deep water wavelength is calculated from:

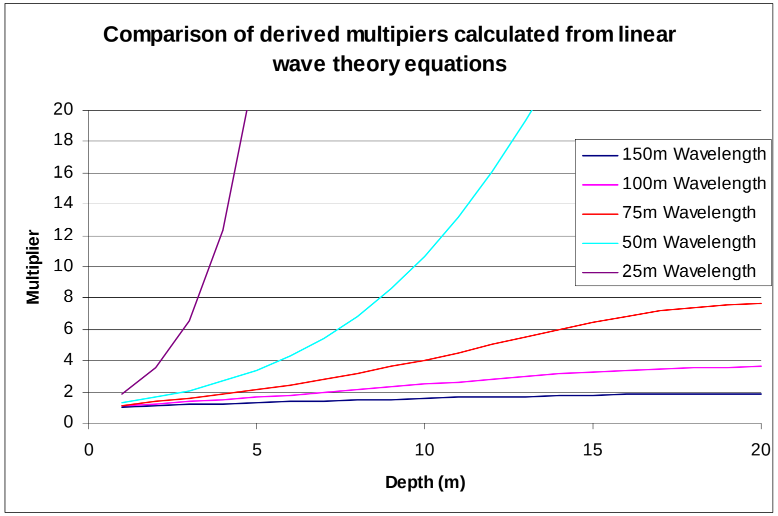
$$L_0 = \frac{gT^2}{2\pi}$$

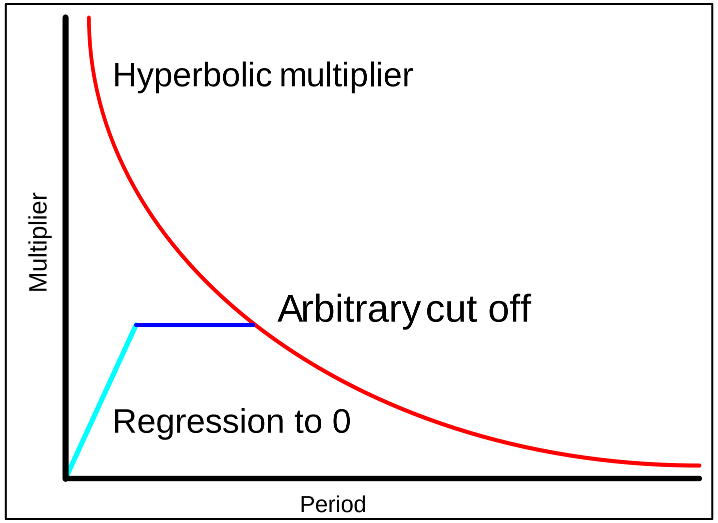
Where:

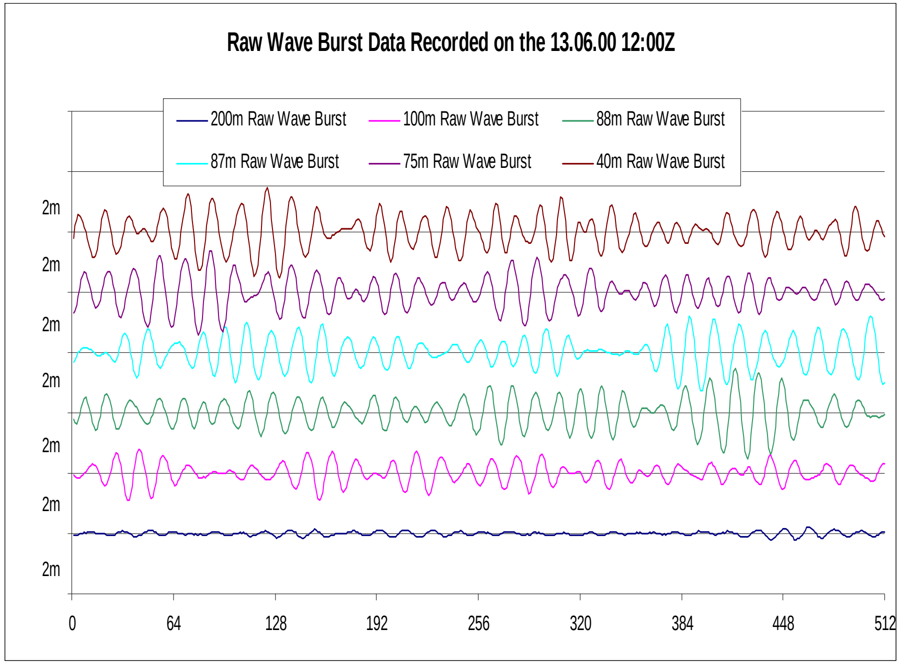
$$L = L_0 \tanh\left(\frac{2\pi}{L}\right)$$

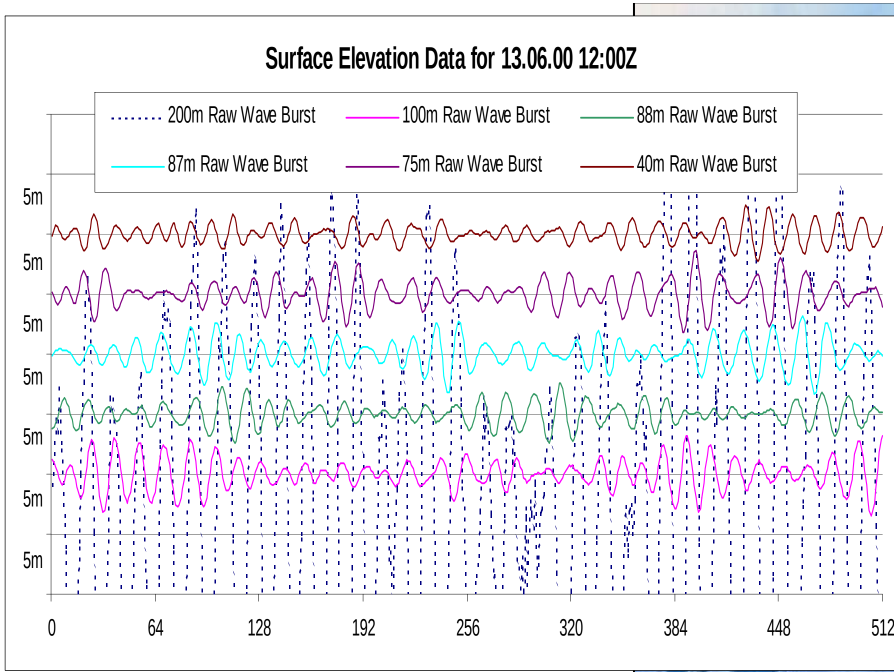
L0 = deep water wavelength
k can now be derived from the relationship k = 2π/L.

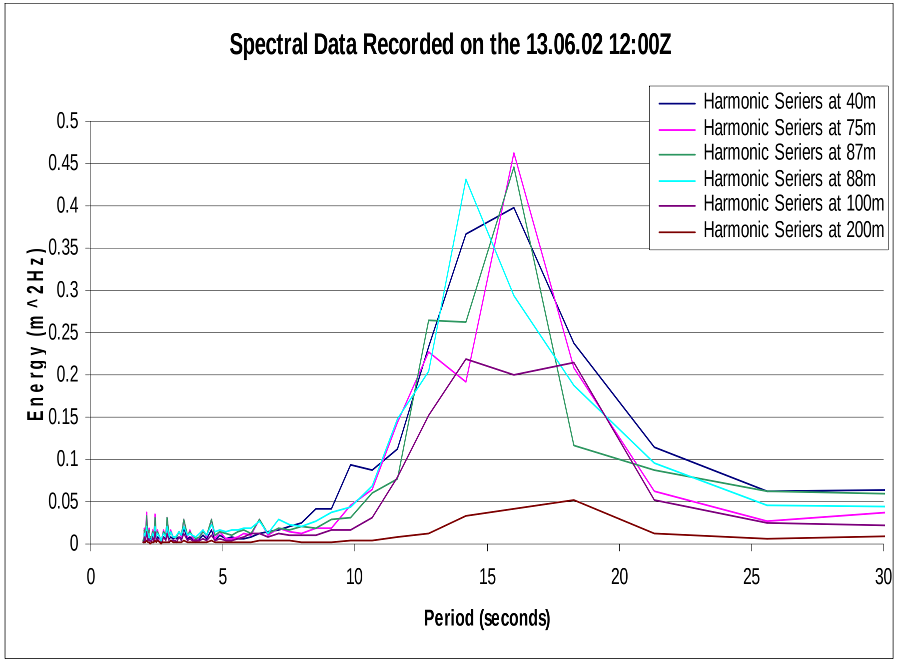
And therefore calculate the attenuation factors,
as these are hyperbolic need an artificial cut off and regression

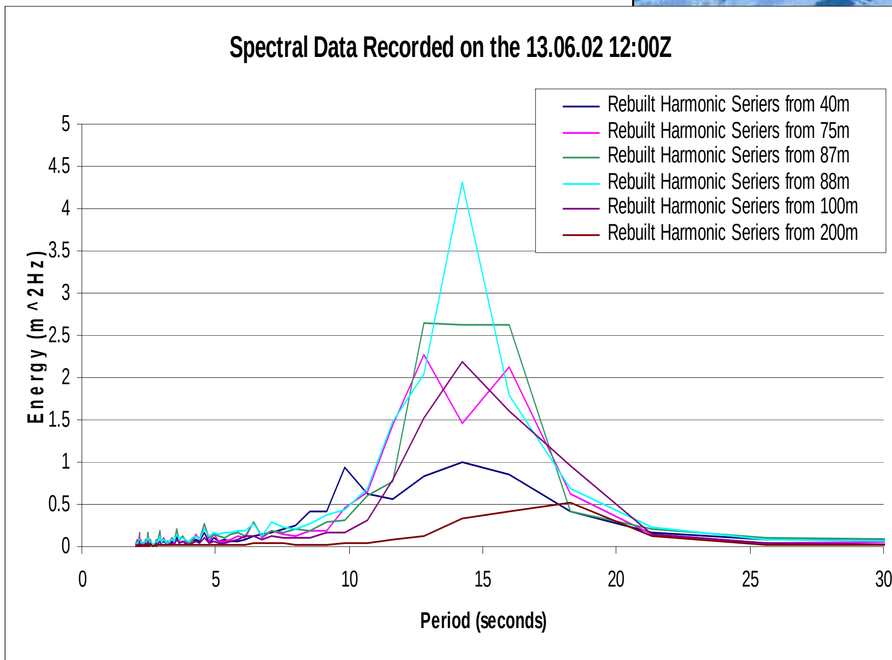


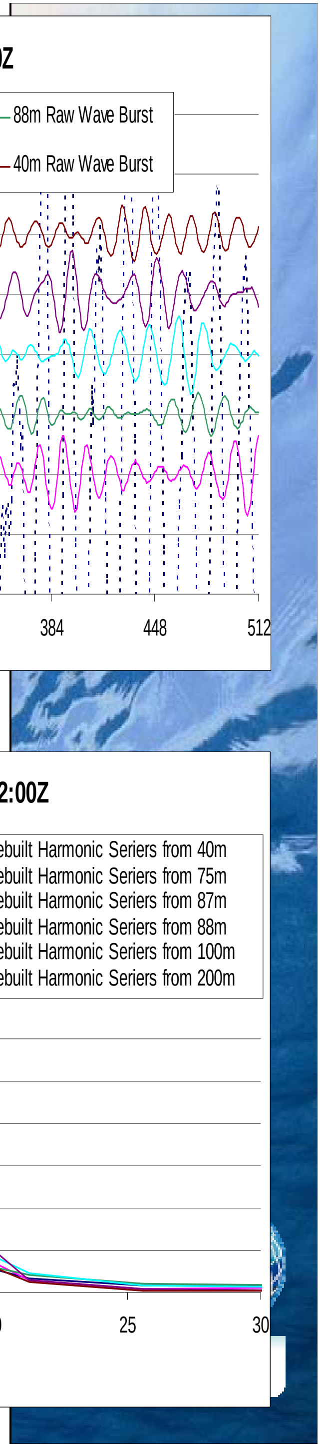



Raw Wave Burst Data Recorded on the 13.06.00 12:00Z
200m Raw Wave Burst
100m Raw Wave Burst
88m Raw Wave Burst
87m Raw Wave Burst
75m Raw Wave Burst
40m Raw Wave Burst
2m
0
64
128
192
256
320
384
448
512


Surface Elevation Data for 13.06.00 12:00Z
200m Raw Wave Burst
100m Raw Wave Burst
88m Raw Wave Burst
87m Raw Wave Burst
75m Raw Wave Burst
40m Raw Wave Burst
5m
0
64
128
192
256
320
384
448
512


Spectral Data Recorded on the 13.06.02 12:00Z
Harmonic Seriers at 40m
Harmonic Seriers at 75m
Harmonic Seriers at 87m
Harmonic Seriers at 88m
Harmonic Seriers at 100m
Harmonic Seriers at 200m
Energy (m ^2Hz)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
Period (seconds)


Spectral Data Recorded on the 13.06.02 12:00Z
Rebuilt Harmonic Seriers from 40m
Rebuilt Harmonic Seriers from 75m
Rebuilt Harmonic Seriers from 87m
Rebuilt Harmonic Seriers from 88m
Rebuilt Harmonic Seriers from 100m
Rebuilt Harmonic Seriers from 200m
Energy (m ^2Hz)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
30
Period (seconds)



Can we use pressure in deep water?

RSN PhD

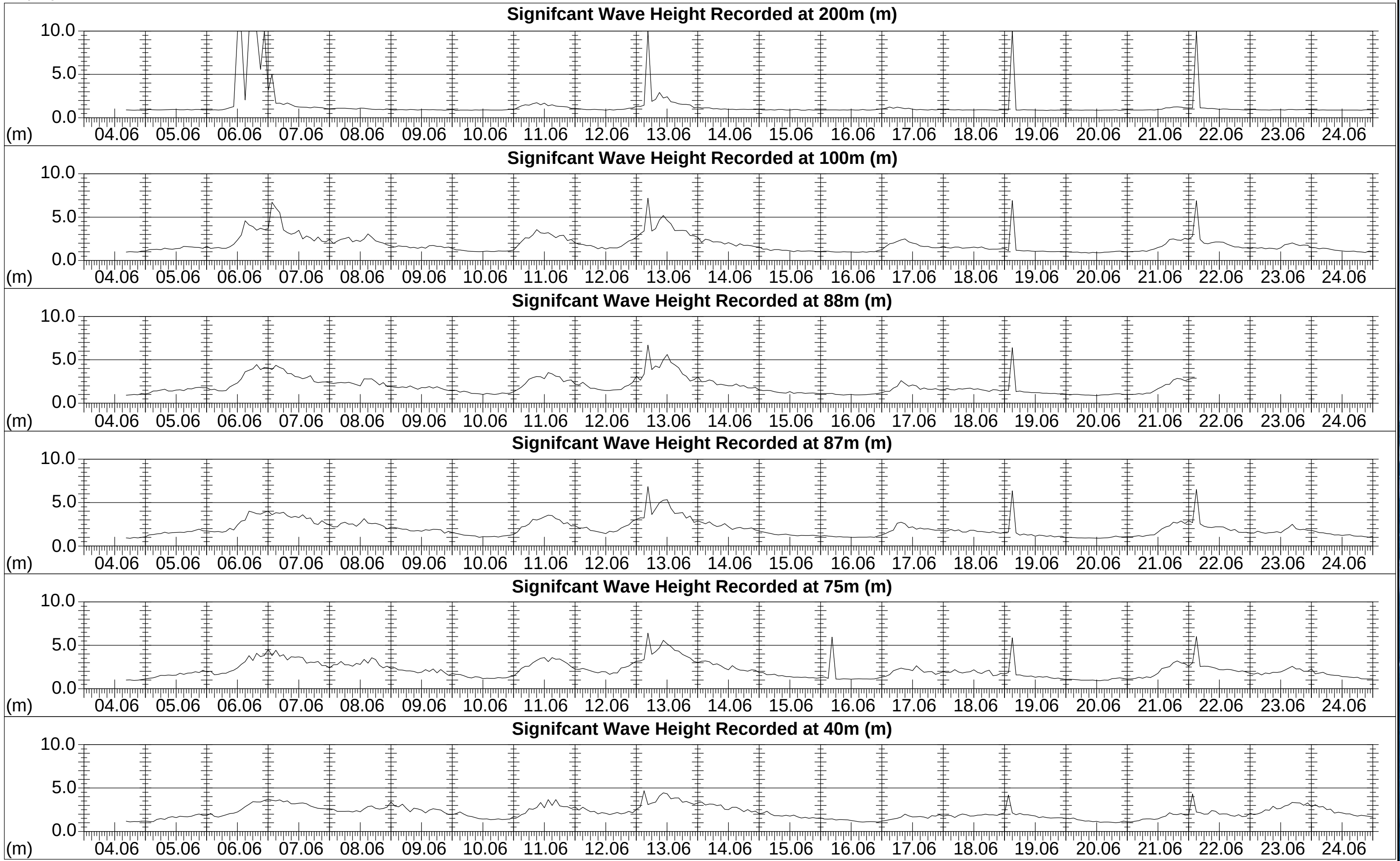
Time Series of Rebuilt Significant Wave Heights

Figure 9

West of Ireland, Atlantic Continental Shelf. Site depths: 40m to 200m Instrument height off seabed: 1.5m

Current Meter / Aquadopp

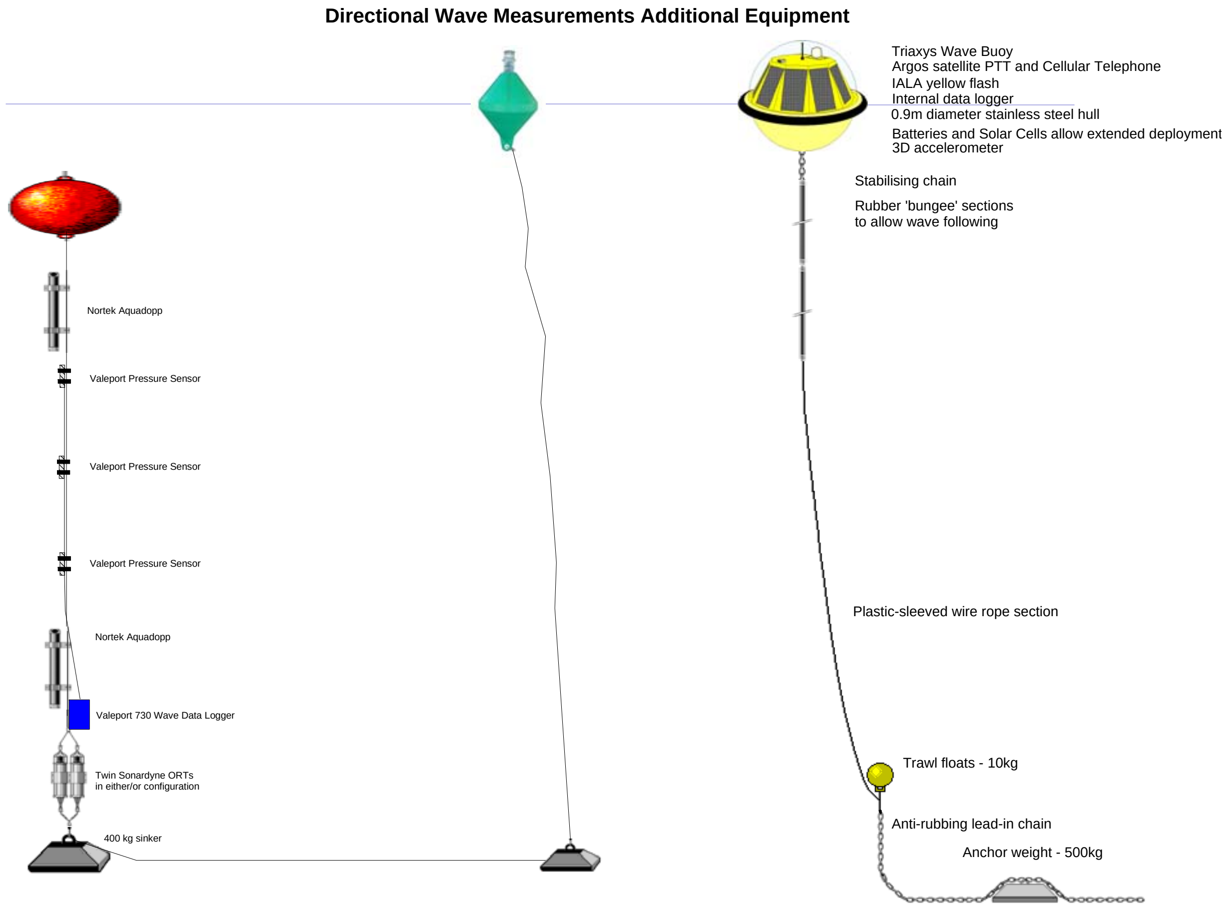
Deployed 04.06.00 Recovered 25.04.00



Wave data can be recorded from a pressure sensor successfully in depths of up to 100m

- Linear wave theory fails to compensate for pressure attenuation
- Long period waves clearly observed in data

Research Deployment:



A comparative deployment was designed using the following:

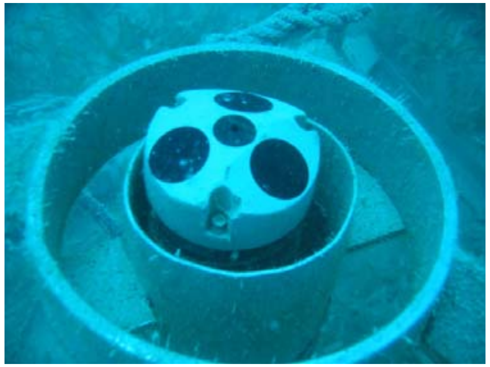
- A surface following wave buoy
- A string of pressure sensors through the water column
- Expanded to include second wave buoy and AWAC
- Deployed within 100m box
- Local data also available from Wave Radar and Step Gauge

Wave Measurement Devices:

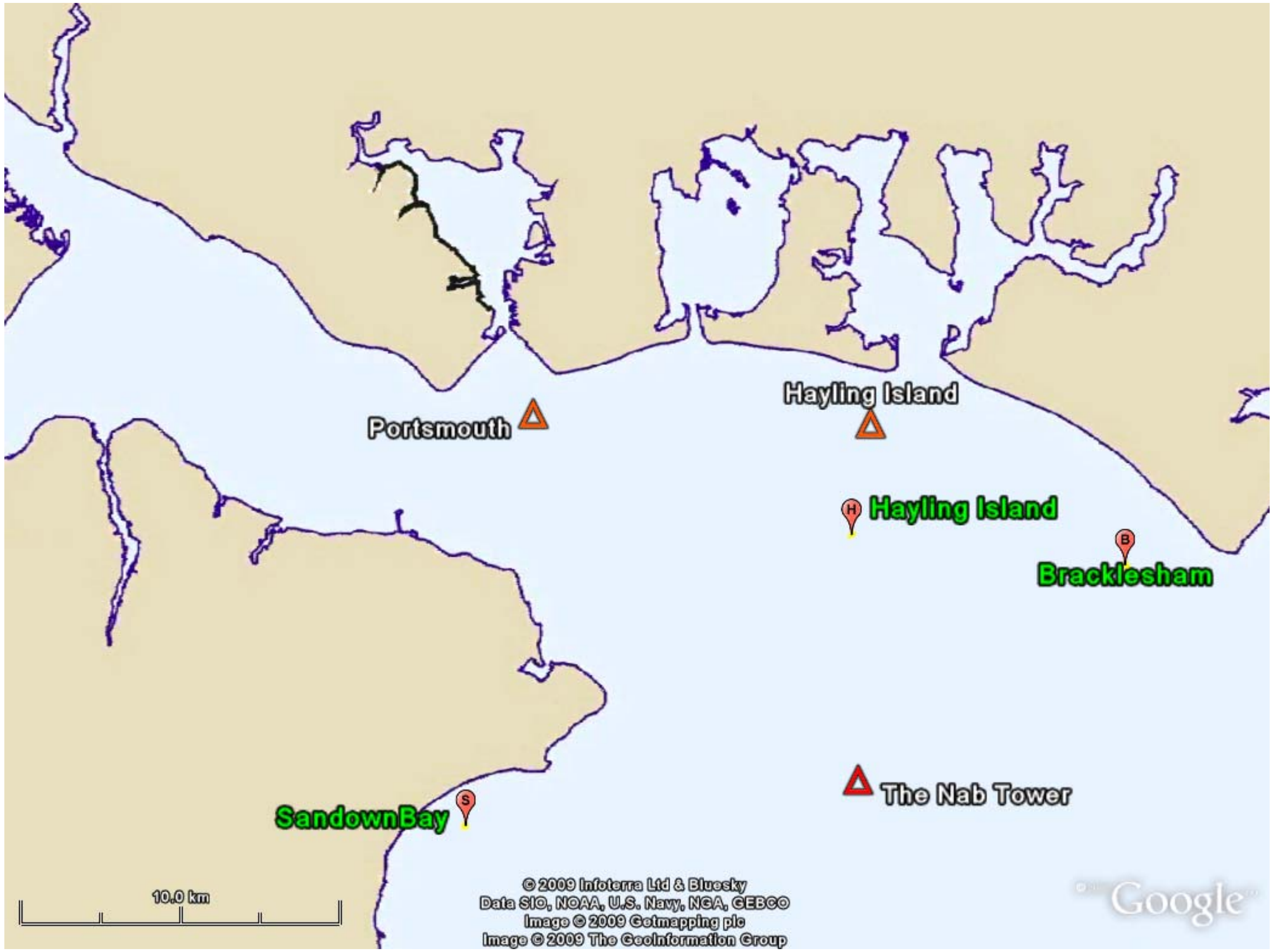
On fixed structures

Surface following buoys

Sub-surface instruments



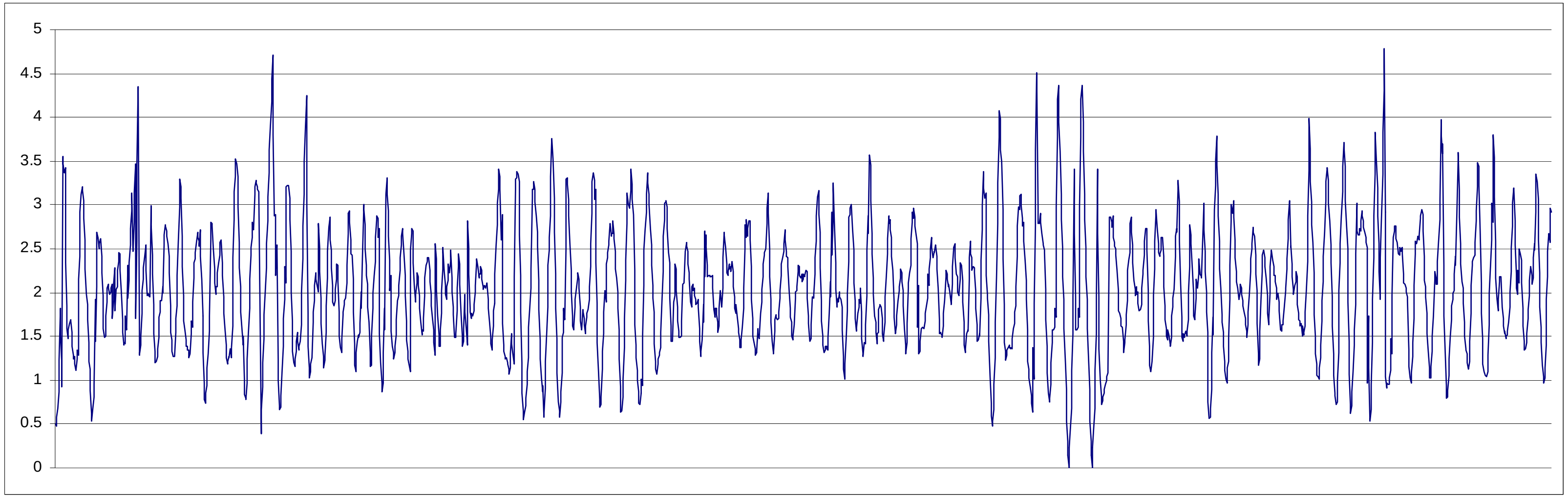
Deployment Location:

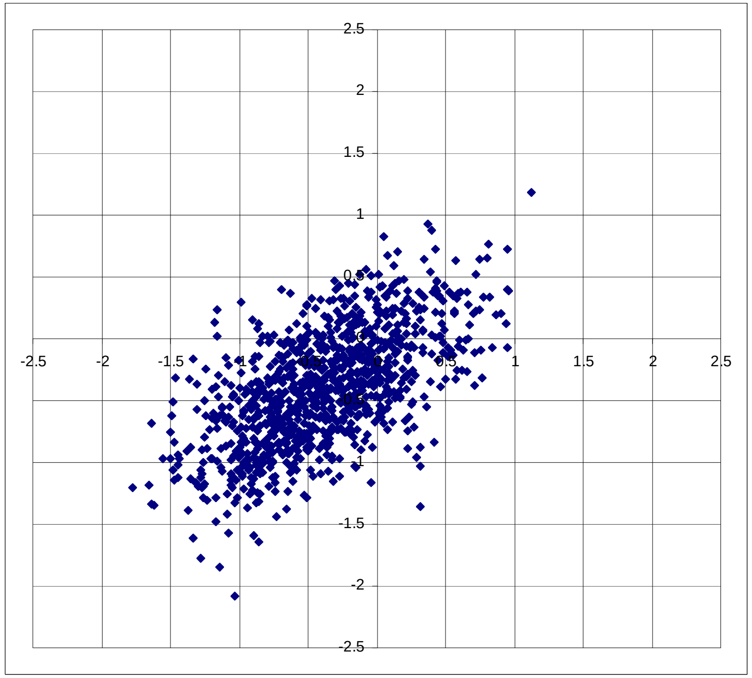


All the key instrumentation were deployed around the Hayling Island wave data site:

- Second research deployment of instruments as part of SemsorGrid4Env
- Next phase – measurements on beach
- Future developments include new instruments on sea defences, bespoke WSN

What is measured?



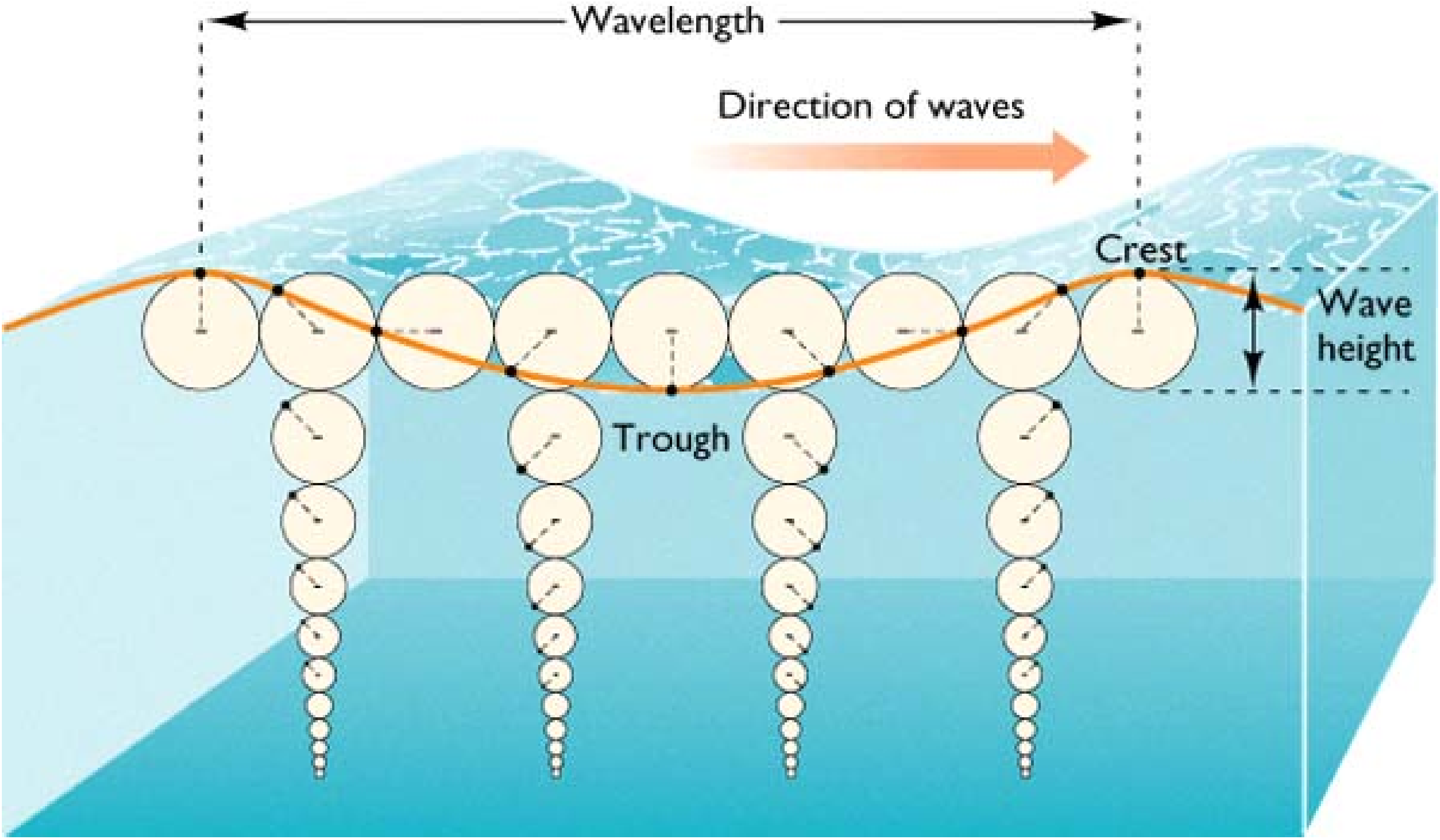


All wave measurements are based on:

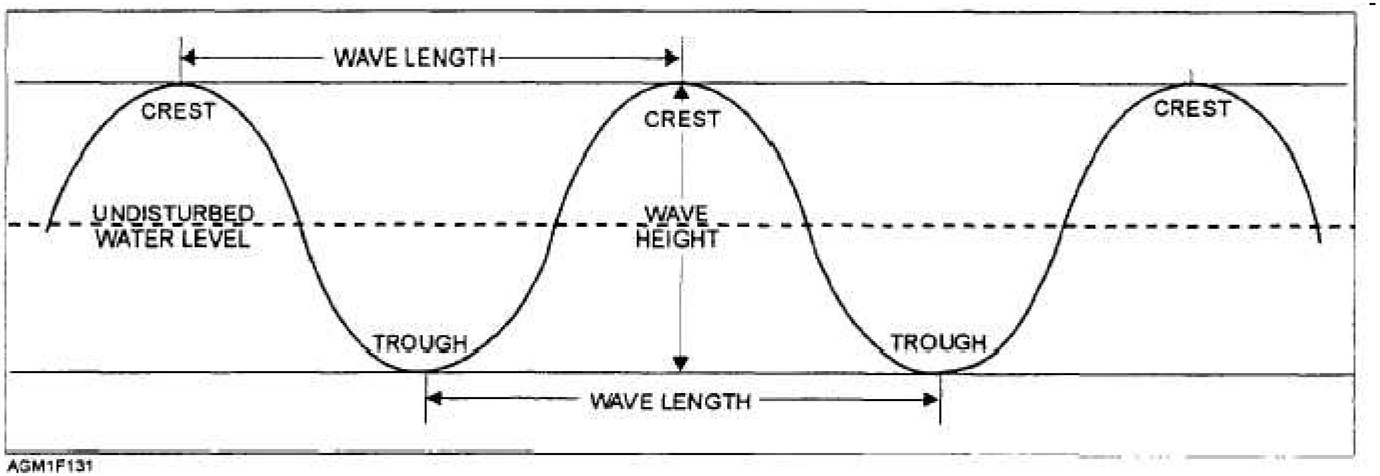
- A time series of surface elevations
- Two sets of horizontal measurements either horizontal accelerations or horizontal current velocity measurements

Everything else is a statistic!

Theoretical wave motion



Theory versus reality

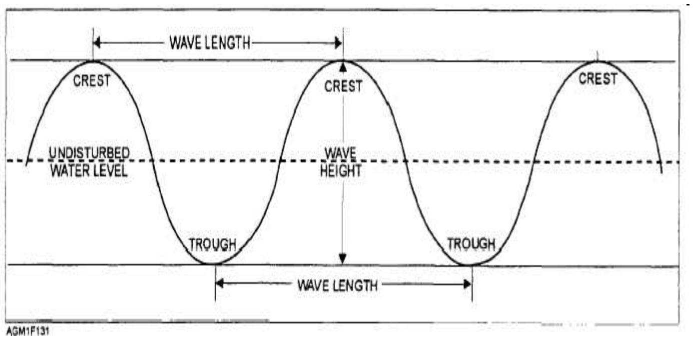


Theory based on sine waves, reality is random wave shapes.
Theory treats crests as continuous, reality is crests are short other than on beaches.

Wave processing approaches

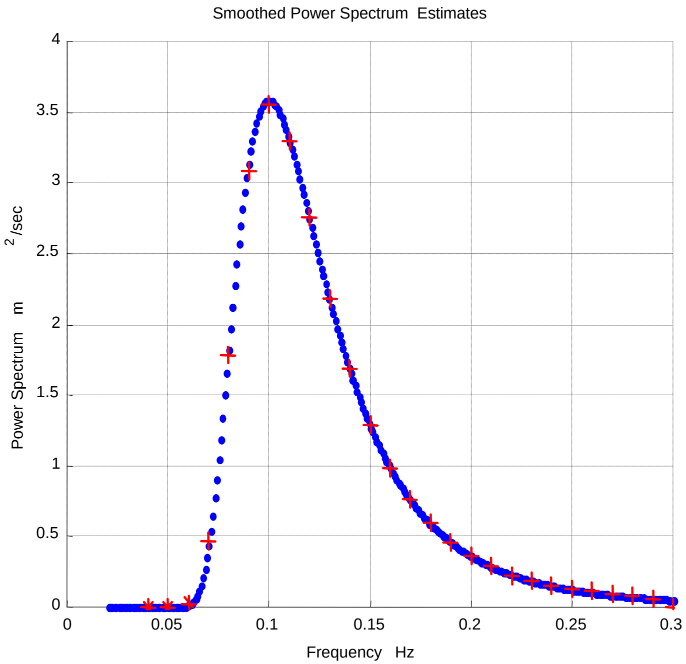
Deterministic processing:

- Parameters derived directly from the time series data for example significant wave height derived from the standard deviation of the surface elevations
- Processing using deterministic approach produces values rapidly and simply
- Very useful for a rapid updating automated system and for verifying spectrally derived parameters



Spectral processing:

- The wave data are passed through a spectral analysis routine to divide the energy into differing periods (or frequencies)
- Industry standard approach based on Fourier Analysis
- Highly versatile – can derive multitude of parameters



Spectral Processing

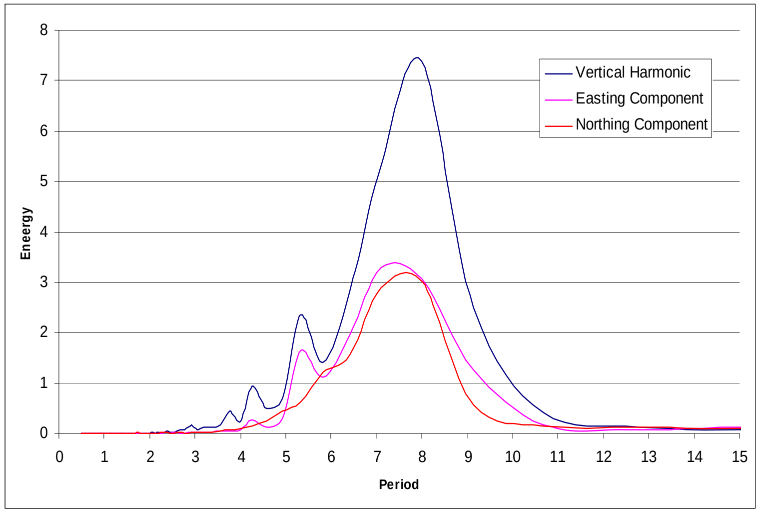
The Maths:

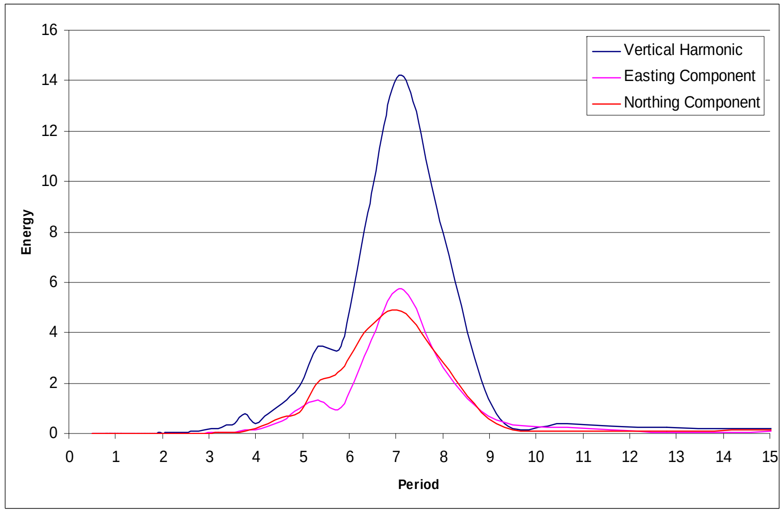
- Wave data broken down into Fourier series

$$\eta(x,\ y,\ t) = \sum_{m=1} \sum_{n=1} A_{mn} \cos\psi_{mn}(x,\ y,\ t)$$ Vertical component

$$u(x,\ y,\ z,\ t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} Q_m(z) \cos\theta_n \cos\psi_{mn}(x,\ y,\ t)$$ First horizontal component

$$v(x,\ y,\ z,\ t) = \sum_{m=1} \sum_{n=1} A_{mn} Q_m(z) \sin\theta_n \cos\psi_{mn}(x,\ y,\ t)$$ Second horizontal component





Next step

To recreate the surface conditions from the pressure data, the linear wave theory equations are used. These are applied directly to the recorded data in addition to the spectral data. The pressure spectrum is related to the surface wave spectrum using the following equations:

$$P_s = \left[\frac{\cosh kh}{\cosh k(h+z)} \right]^2 \frac{P_d}{\rho^2 g^2}$$

Ps = pressure signal at surface
Pd = pressure signal at sensor
k = wave number
h = mean depth of burst
z = sensor height above seabed
ρ = water density (not required if values previously converted to depths)
g = local gravity (not required if values previously converted to depths)

The wave number (k) is derived from the following equations:

$$k = \frac{2\pi}{L}$$

c² = gk tanh kh

$$c = \frac{L}{T}$$

Where: c = wave speed (celerity)
k = wave number
h = mean depth of burst
g = local gravity (not required if values previously converted to depths)

Using the relationships:

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi}{L} \right)$$

and

$$L_0 = \frac{gT^2}{2\pi}$$

the derivation of L can be used.

$$L = L_0 \tanh\left(\frac{2\pi}{L} \right)$$

L = wavelength
T = wave period

To finally derive the spectral moments

$$m_n = \sum_z^1 \left[f^i E(f_i) \Delta f_i \right]$$

•calculated significant wave (Hs) = 4 * $m_0^{½}$, where mo (zeroth moment) is the variance and $\sqrt{m_0}$ is the standard deviation
•calculated mean period T_1 = defined as m_o/m_1
•calculated zero-upcrossing period (Tz) = defined as $(m_0/m_2)^{½}$
•peak period (Tp) = taken from spectral frequency bands
•calculated total energy (Et) = total energy by density * gravity * variance (m_o)
•calculated significant wave period (Tsig) = the average period of the waves defined by the significant wave height

And then...

To define the spectrally derived peak coming direction, the standard approach is to calculate the weighted mean coming direction for each spectral period. This is expressed by the following relationship:

$$\theta_f = \arctan 2\left(a_1{}^2 + b_1{}^2\right) \qquad a_1 = \frac{C_{zu}}{\sqrt{C_{zz}(C_{uu}+C_{vv})}} \qquad b_1 = \frac{C_{zv}}{\sqrt{C_{zz}(C_{uu}+C_{vv})}}$$

Where:
Czz = Co-spectra of the vertical displacements
Cuu = Co-spectra of the easting displacements
Cvv = Co-spectra of the northing displacements
Czu = Cross-spectra of the vertical and northing displacements
Czv = Cross-spectra of the vertical and northing displacements

$$C_{zz} = \alpha_{zf}{}^2 + \beta_{zf}{}^2 \qquad C_{vz} = \alpha_{vf}\beta_{zf} - \beta_{vf}\alpha_{zf}$$

Where:
alpha = The sine component of the spectral analysis
beta = The cosine component of the spectral analysis

The spectral energy can be summarised by the following relationship:

E(f,θ) = S(f) D(f,θ)

Where: E(f,θ) = the energy in $m^2Hz/°$ for each combination of direction and frequency

S(f) = 1 dimensional spectral energy at the frequency f

And D(f,θ) is described by the following relationship:

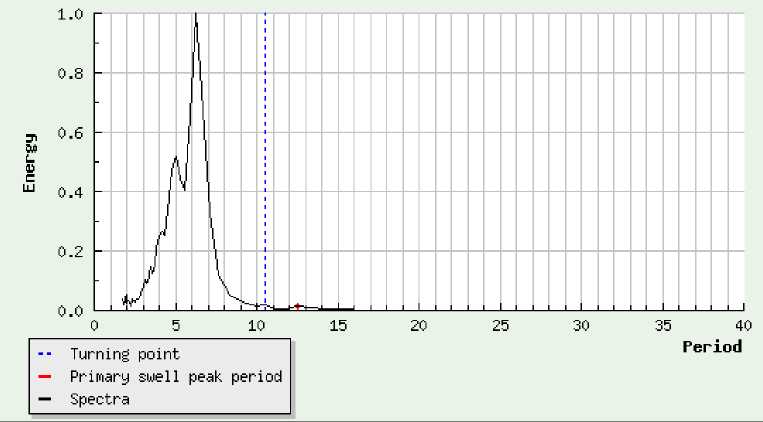
$$\mathrm{D}(\mathrm{f},\theta) = \frac{1}{\pi}\left(\frac{1}{2} + a_1\cos\theta + b_1\sin\theta + a_2\cos 2\theta + b_2\sin 2\theta\right)$$

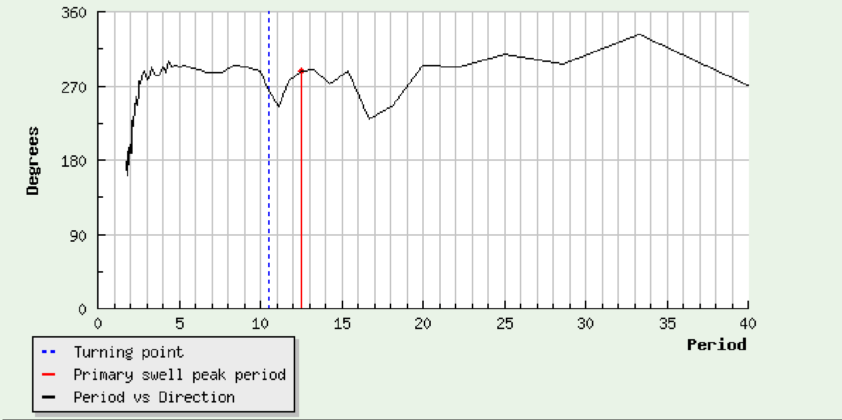
$$a_2 = \frac{C_{uu} - C_{zz}}{C_{uu} + C_{zz}} \qquad b_2 = \frac{-2C_{zv}}{C_{zz} + C_{vv}}$$

Simple...

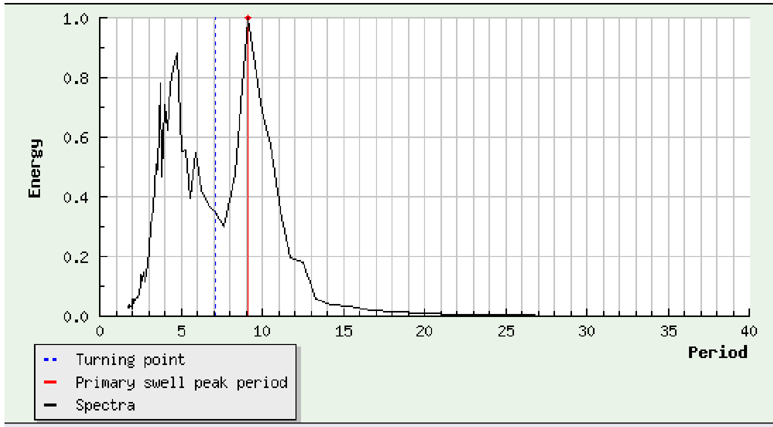
Example spectra...

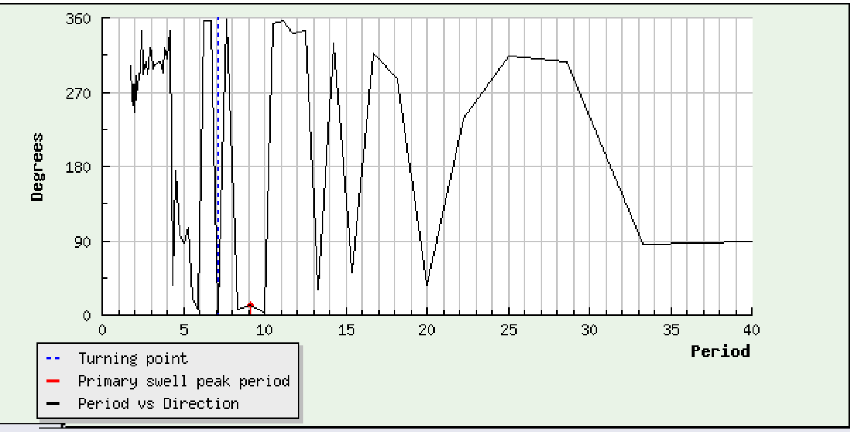
Monomodal:

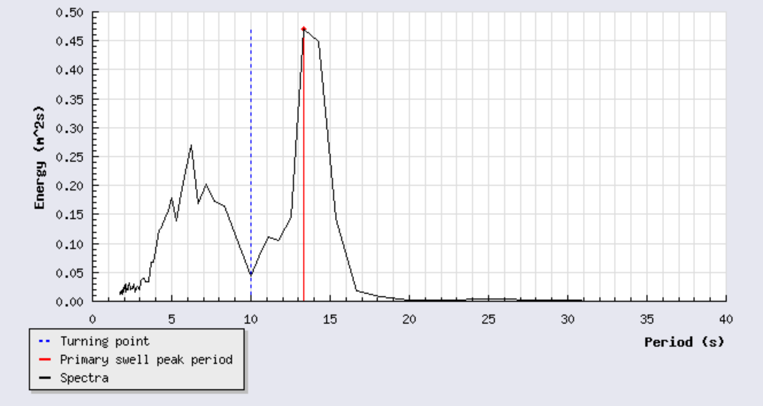


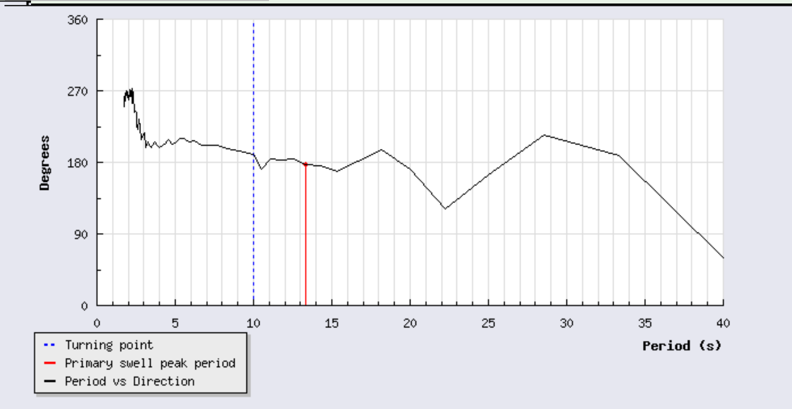


Bimodal:









2 D wave spectra…

Client

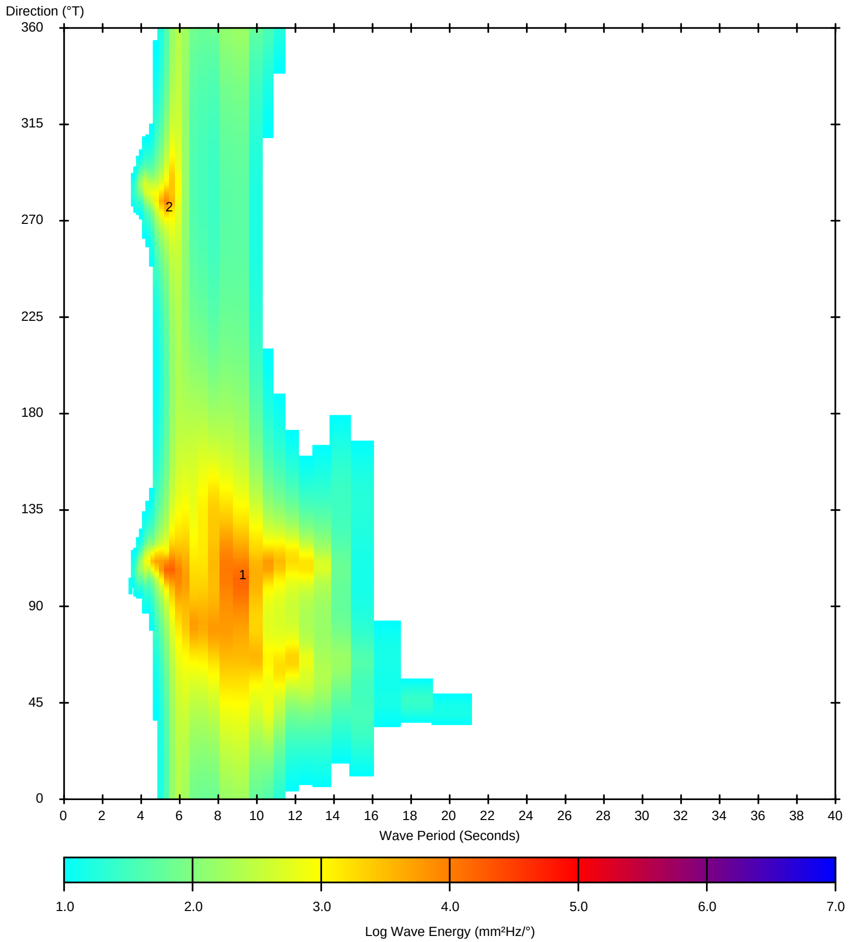
Directional Wave Spectral Analysis
Job Name

Figure 3801

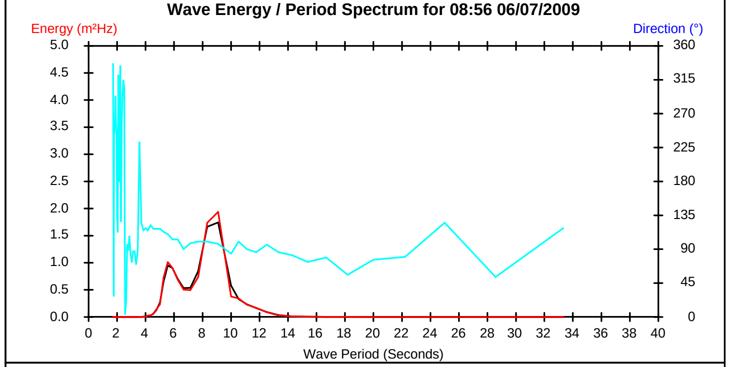
Position: Latitude: XX° XX.XXX' N, Longitude: XXX° XX.XXX' W Site Depth: 100m
Deployed: 05:25 19/04/2009, Recovered 06/08/2009

Datawell DWR MkIII
Serial No.: 30554

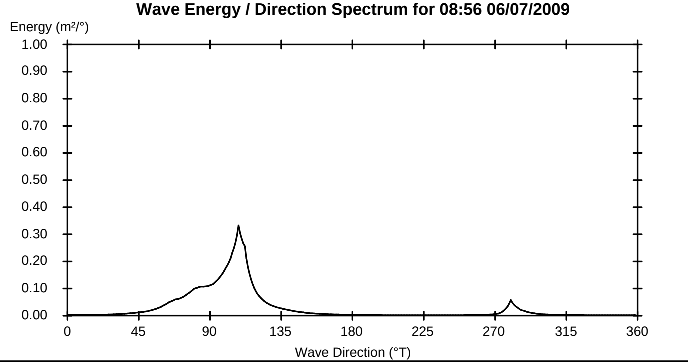
Two Dimensional Wave Energy Spectrum for 08:56 06/07/2009



Wave Energy / Period Spectrum for 08:56 06/07/2009



Wave Energy / Direction Spectrum for 08:56 06/07/2009



Wave Statistics for 08:56 06/07/2009

Zeroth Spectral Moment: 0.052m²
Significant Wave Height: 0.92m
Maximum Wave Height: 1.25m

Peak Period: 9.1s
Zero Up Crossing Period: 6.8s
Mean Coming Direction (°T): 76.8°
Peak Coming Direction (°T): 97.0°
Directional - Peak Direction (°T): 108.0°

Primary Zeroth Spectral Moment: 0.048m²
Primary Significant Wave Height: 0.88m
Primary Peak Period: 9.1s
Primary Peak Direction: 108°

Secondary Zeroth Spectral Moment: 0.003m²
Secondary Significant Wave Height: 0.23m
Secondary Peak Period: 5.3s
Secondary Peak Direction: 280°

Emu Limited Oceanographic Section

Job No: J/1/01/XXXX

Barrier breaching models – use H_s & T_m

Hayling Island, 03 November 2005

Photo: Clive Moon, Havant Borough Council

•Wave measurement: is a pressure sensor ever the right choice? Yes…

Wave data can be recorded from a pressure sensor successfully in depths of up to 100m

- We need to improve the mathematical algorithms that compensate for the pressure attenuation
- Pressure sensors can be the only option for instance measuring run up on a beach
- Works well for long period waves
- To be continued…


Any Questions?