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3. Theoretical Background

Spectrum Width

Spectral width is quantified by the normalized radius of gyration, ν , which describes the way in which spectral area is distributed about the mean frequency:

$$\nu = \frac{\int_0^\infty f^2 E(f) df}{m_0} - 1 \quad (\text{Eq. 1})$$

where m_n is the n -th spectral moment: $m_n = \int_0^\infty f^n E(f) df$ (Eq. 2)

and $E(f)$ is the spectral energy density; m_0 is the sea surface variance. For very narrow bandwidths, ν goes to zero and all wave energy is concentrated near one frequency. Larger values of ν are associated with wide spectra, when wave energy is broadly distributed among frequencies or multiple peaks are present.

Narrow: Waves have almost the same frequency with gradually varying amplitudes modulated by the wave envelope. The upper and lower envelopes coincide with crests and troughs and form a pair of symmetric curves. Individual wave heights are approximately equal to twice the wave amplitude (Fig. 1a).

Wide: Broad distribution of energy and waves of many frequencies are present. Waves ride on each other to produce local maxima both above and below the mean sea level (Fig. 1b).

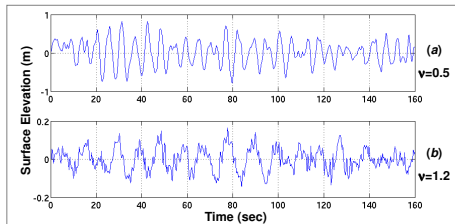


Figure 1. Sample time histories for (a) narrow and (b) wide spectra.

Significant Wave Height

Longuet-Higgins (1952) first applied the statistical theory of random signals to narrow-band ocean waves and demonstrated that wave heights possess a Rayleigh distribution. From the Rayleigh distribution it can be shown that for narrow-band spectra:

$$H_{1/3} = 1.416 H_{rms} = (1.416)(2\sqrt{2}m_0) = 4.005 \sqrt{m_0} = H_{m_0} \quad (\text{Eq. 3})$$

Historically, the "significant wave height" was estimated visually by trained observers. $H_{1/3}$ is the average of the 1/3 highest waves, was chosen to best match the historical definition and is estimated through zero-crossing analysis of the surface elevation time series. Spectral analysis gives an energy-based significant wave height, H_{m_0} , which is roughly four times the square root of the sea surface variance, m_0 (Eq. 3). As a rule of thumb, Tucker and Pitt (2001) suggest that $H_{1/3}$ is typically 90-100% of H_{m_0} ($1 < H_{m_0}/H_{1/3} < 1.11$).

Maximum Wave Height

The maximum wave height depends fundamentally on the number of waves in the sample, N . For each burst, the ratio $H_{max}/H_{1/3}$ can be treated as a random variable, and there will be a corresponding probability distribution that yields the expected value of the ratio. Longuet-Higgins (1952) provides a formulation for estimating this ratio given ρ and N :

$$\frac{E(H_{max})}{H_p} = \frac{E(H_{max})/H}{E(H_{max})/H} \sqrt{\frac{\log \frac{1}{\rho} + \frac{1}{2} \sqrt{\frac{2}{\pi}} \text{erfc} \left(\sqrt{\log \frac{1}{\rho}} \right)}{\log \frac{1}{\rho} + \frac{1}{2} \gamma (\log N)^{1/2}}} \quad (\text{Eq. 4})$$

where $\gamma=0.5772$; H_p is the average of the highest pN waves, where $0 < \rho \leq 1$. For significant wave height ($H_{1/3}$) $\rho=1/3$.

References

Longuet-Higgins, M.S. (1952). "On the statistical distribution of the heights of sea waves." *Journal of Marine Research*. Vol. XI, No. 3, pp. 245-266.
 Tucker, M.J., and E.G. Pitt (2001). *Waves in Ocean Engineering*. Elsevier Ocean Engineering Book Series, Vol. 5.

1. Abstract

Accurate estimates of wave height and period in real-time operational deployments and modeling studies are becoming increasingly important in the coastal environment. Zero-crossing analysis and spectral methods have historically been used to estimate wave parameters such as $H_{1/3}$, H_{m_0} , $H_{1/10}$, H_{max} , T_p , and $T_{0.5}$. In this study, we present comparisons of wave parameter estimates by both methods using data from acoustic Doppler wave gauges in nine environments ranging from fetch-limited estuarine systems to high-energy exposed coasts. The results suggest that the agreement between the methods (H_{m_0} vs. $H_{1/3}$) is linked to the underlying theoretical assumption of a narrow-banded spectrum and a divergence from theory occurs as spectral width increases with changes in energy regime and wave-current interaction. Examination of the $H_{max}/H_{1/3}$ ratio shows a large random variation that can be difficult to predict analytically on burst-to-burst time scales; however, long-term measurements show a predictable dependence on the site-specific wave climate.

4. Results

Significant Wave Height

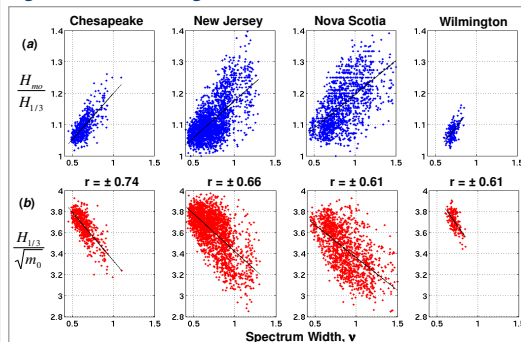


Figure 2. (a) $H_{max}/H_{1/3}$ is positively correlated with the spectrum bandwidth parameter at all sites. The strongest correlations are shown above. (b) Agreement between wave height estimates is evaluated by solving for the ν coefficient in Eq. 3. The observed value approaches 4.0 as the narrow-band assumption is satisfied but deviates by as much as 25% as spectrum width increases.

Maximum Wave Height

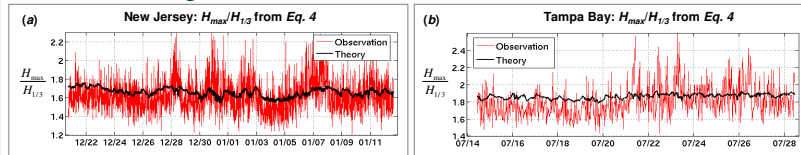


Figure 4. $H_{max}/H_{1/3}$ shows large random variation from burst-to-burst about the theoretical value (Eq. 4); however, long-term measurements show a predictable dependence on the site-specific wave climate at (a) New Jersey, (b) Tampa Bay.

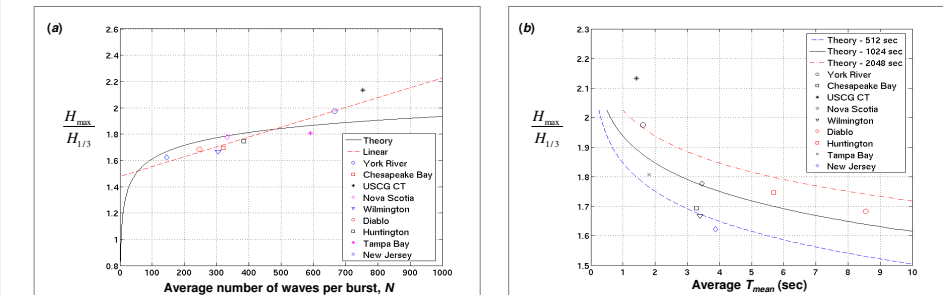
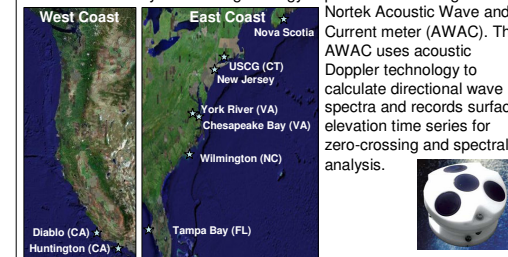


Figure 5. (a) Comparison of the theoretical curve (black) vs. mean observed values of N and $H_{max}/H_{1/3}$ at each site (Note: N =burst length/ T_{mean}). The data agree favorably and tend to show the general increase of $H_{max}/H_{1/3}$ with increasing N . While the theory indicates a logarithmic dependence on N , a linear trend (red) fits the observations quite well ($r=0.88$). (b) Comparison of the theoretical curves vs. mean observed values of T_{mean} and $H_{max}/H_{1/3}$. The curves for each burst length (512 sec-blue, 1024 sec-black, 2048 sec-red) correspond to the symbols of like color.

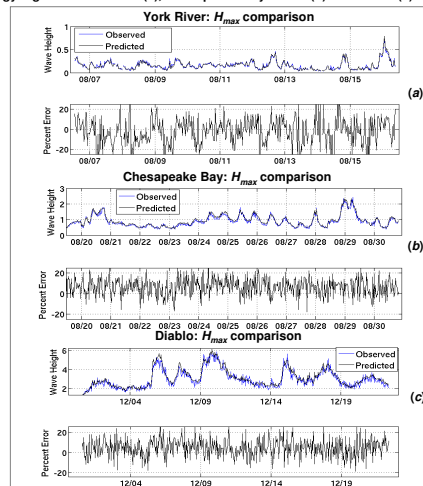
2. Study Sites

Wave and current data were collected at nine sites ranging from fetch-limited estuarine systems to high-energy exposed coasts using the Nortek Acoustic Wave and Current meter (AWAC). The AWAC uses acoustic Doppler technology to calculate directional wave spectra and records surface elevation time series for zero-crossing and spectral analysis.



Predicting H_{max}

Figure 6. H_{max} can be predicted from Eq. 4 using T_{mean} as a proxy for N (T_{mean} is calculated from spectral moments). The observed $H_{1/3}$ is then multiplied by the theoretical ratio to obtain H_{max} . The figures below show the agreement for three energy regimes: York River (a), Chesapeake Bay Mouth (b) and Diablo (c).



5. Conclusions

- $H_{m_0}/H_{1/3}$, $H_{1/3}/\sqrt{m_0}$, and ν vary at synoptic time scales with changes in energy regime and spectrum shape.
- $H_{m_0}/H_{1/3}$ is positively correlated with ν for the Chesapeake (0.74), New Jersey (0.66), Nova Scotia (0.61), Wilmington (0.61), and Tampa Bay (0.43) sites (Fig. 2a). Other sites showed weak positive correlations ($r < 0.40$).
- $H_{1/3}/\sqrt{m_0}$ is negatively correlated with ν at all sites and approaches the theoretical value of 4.0 (Eq. 3) for narrow bandwidths, but diverges from theory by up to 25% as spectrum width increases.
- $H_{max}/H_{1/3}$ shows large random variation from burst to burst and a more gradual variation at synoptic time scales at all sites (Fig. 4), but displayed no clear dependence on ν .
- At each site, the mean observed $H_{max}/H_{1/3}$ agrees favorably with the theoretical prediction from Eq. 4 using the mean observed N (Fig. 5).
- Given an accurate estimate of significant wave height, H_{max} can be predicted analytically from Eq. 4 within an error of $\pm 20\%$ (Fig. 6).

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